APPLICATION NO. 09/826,117

TITLE OF INVENTION: Hyhrid Walsh Codes for CDMA

INVENTOR: Urbain A. von der Embse

Marked up version of SUBSTITUTE SPECIFICATION using the original specification of 2001

APPLICATION NO. 09/826,117

TITLE OF INVENTION: Hybrid Complex Walsh Codes for CDMA

INVENTORS: Urbain ~~Alfred~~ A. von der Embse

BACKGROUND OF THE INVENTION

I. Field of the Invention

~~**TECHNICAL FIELD**~~

The present invention relates to CDMA (Code Division Multiple Access) cellular telephone and wireless data communications with data rates up to multiple T1 (1.544 Mbps) and higher (>100 Mbps), and to optical CDMA with data rates in the Gbps and higher ranges. Applications are mobile, point-to-point and satellite communication networks. More specifically the present invention relates to novel complex and ~~hybrid~~ generalized complex Walsh codes developed to replace current ~~real~~ Walsh orthogonal CDMA channelization codes which are real Walsh codes.

~~**CONTENTS**~~

II. Description of the Related Art

~~**BACKGROUND ART**~~

Current CDMA art is represented by the recent work on multiple access for broadband wireless communications, the G3 (third generation CDMA) proposed standard candidates, the current IS-95 CDMA standard, the early Qualcomm patents, and the real Walsh technology. These are documented in ~~references 1,2,3,4,5,6. Reference 1 is an issue of the IEEE communications magazine devoted to multiple access communications for broadband wireless networks, reference 2 is an issue on IEEE personal communications devoted to the third generation (3G) mobile systems in Europe~~"Multiple Access for Broadband Networks", IEEE Communications magazine July 2000 Vol. 38 No. 7, "Third Generation Mobile Systems in Europe", IEEE Personal Communications April 1998 Vol. 5 No. 2, ~~reference 3 is the~~ IS-95/IS-95A, ~~standard primarily developed by Qualcomm, references 4 and 5 are Qualcomm patents addressing the use of real Walsh orthogonal CDMA codes, and reference 6 is the widely used reference on real Walsh technology.~~ the IS-95/IS-95A, the 3G CDMA2000 and W-CDMA, and the listed patents.

Current art using ~~real~~ Walsh orthogonal CDMA channelization codes is represented by the scenario described in the following with the aid of equations **(1)** and FIG **1,2,3,4**. This scenario considers CDMA communications spread over a common frequency band for each of the communication channels with each channel defined by a CDMA code. These CDMA communications channels for each of the users are defined by assigning a unique Walsh orthogonal spreading codes to each user. The Walsh code for each user spreads the user data symbols over the common frequency band. These Walsh encoded user signals are summed and re-spread over the same frequency band by ~~one or more~~long and short pseudo-noise PN codes, to generate the CDMA communications signal which is

modulated and transmitted. The communications link consists of a transmitter, propagation path, and receiver, as well as interfaces and control.

It is assumed that the communication link is in the communications mode with all of the users communicating at the same symbol rate and the synchronization is sufficiently accurate and robust to support this communications mode. In addition, the possible power differences between the users is assumed to be incorporated in the data symbol amplitudes prior to the CDMA encoding in the CDMA transmitter, and the power is uniformly spread over the wideband by proper selection of the CDMA pulse waveform. ~~It is self evident to anyone skilled in the CDMA communications art that these communications mode assumptions are both reasonable and representative of the current CDMA art and do not limit the applicability of this invention.~~

Transmitter equations **(1)** describe a representative real Walsh CDMA encoding for the transmitter in FIG. **1**. It is assumed that there are N Walsh code vectors W(u) each of length N chips **1**. The code vector is presented by a 1xN N-chip row vector W(u)=[W(u,1),…,W(u,N)] where W(u,n) is chip n of code u. The code vectors are the row vectors of the Walsh matrix W. Walsh code chip n of code vector u has the possible values W(u,n)= +/-1. Each user is assigned a unique Walsh code which allows the code vectors to be designated by the user symbols u=0,1,…,N-1 for N Walsh codes. User data symbols **2** are the set of complex symbols {Z(u), u=0,,1,…,N-1) and the set of real symbols (R(u_R), I($\underline{u}_I$), u_R, u_I =0,1,…,N-1} where Z is a complex symbol and R,I are real symbols assigned to the real, imaginary communications axis. Examples of complex user symbols are QPSK and OQPSK encoded data corresponding to 4-phase and offset 4-phase symbol coding. Examples of real user symbols are PSK and DPSK encoded data corresponding to 2-phase and differential 2-phase symbol coding. Although not considered in this example, it

is possible to use combinations of both complex and real data symbols.

Current real Walsh CDMA encoding for transmitter **(1)**

1 Walsh codes

W = Walsh NxN orthogonal code matrix consisting of N rows of N chip code vectors

= [W(u)] matrix of row vectors W(u)

= [W(u,n)] matrix of elements W(u,n)

W(u) = Walsh code vector u for u=0,1,…,N-1

= [W(u,0), W(u,1), …, W(u,N-1)]

= 1xN row vector of chips W(u,0),….,W(u,N-1)

W(u,n) = Walsh code u chip n

= +/+1 possible values

2 Data symbols

Z(u) = Complex data symbol for user u

$R(u_R)$ = Real data symbol for user u_R assigned to the Real axis of the CDMA signal

$I(u_I)$ = Real data symbol for user u_I assigned to the Imaginary axis of the CDMA signal

3 Walsh encoded data

Complex data symbols

Z(u,n) = Z(u) sgn{ W(u,n)}

= User u chip n Walsh encoded complex data

Real data symbols

$R(u_R,n)$ = $R(u_R)$ sgn{ $W(u_R,n)$ }

= User u_R chip n Walsh encoded real data

$I(u_I,n)$ = $R(u_R)$ sgn{ $W(u_R,n)$ }

= User u_I chip n Walsh encoded real data

where sgn{ (o) } = Algeraic sign of "(o)"

4 ~~4~~ PN scrambling

$P_2(n), P_{R2}(n), P_{I2}(n)$ = Chip n of long PN codes

$P_R(n)$ = Chip n of short PN code~~s~~ for real axis

$P_I(n)$ = Chip n of short PN code~~s~~ for imaginary ~~Axis~~axis

Complex data symbols:

$$Z(n) = \text{PN scrambled real Walsh encoded data chips after summing over the users}$$

$$= \sum_u \mathbf{Z(u,n)} \mathbf{P}_2(n)\,[\mathbf{P}_R(n) + j\,\mathbf{P}_I(n)]$$

$$= \sum_u \mathbf{Z(u,n)}\text{sign}\{P_2(n)\}\,[\text{sgn}\{P_R(n)\} + j\,\text{sgn}\{P_I(n)\}]$$

$$= \text{Real Walsh CDMA encoded complex chips}$$

Real data symbols:

$$Z(n) =$$

$$\Big[\sum_{u_R} \mathbf{R}(\mathbf{u}_R, \mathbf{n})\text{sgn}\{P_{R2}(n)\} + \mathbf{j}\sum_{u_I} \mathbf{I}(\mathbf{u}_I, \mathbf{n})\Big]\text{sgn}\{P_{I2}(n)\} \quad [\text{sign}\{P_R(n)\} + j\ \text{sign}\{P_I(n)\}]$$

$$= \text{Real Walsh CDMA encoded real chips}$$

User data is encoded by the Walsh CDMA codes **3.** Each of the user symbols $Z(u), R(u_R), I(u_I)$ is assigned a unique Walsh code. $W(u), W(u_R), W(u_I)$. Walsh encoding of each user data symbol generates an N-chip sequence with each chip in the sequence consisting of the user data symbol with the sign of the corresponding Walsh code chip, which means each chip = [Data symbol] x [Sign of Walsh chip].

The Walsh encoded data symbols are summed and encoded with PN codes **4.** These long PN codes are 2-phase with each chip equal to +/-1 which means PN encoding consists of sign changes with each sign change corresponding to the sign of the PN chip. Short PN codes are complex with 2-phase codes along their real

and imaginary axes. Encoding with a long PN means each chip of the summed Walsh encoded data symbols has a sign change when the corresponding long PN chip is -1, and remains unchanged for +1 values. This operation is described by a multiplication of each chip of the summed Walsh encoded data symbols with the sign of the PN chip. Purpose of the PN encoding ~~for complex data symbols~~ is to provide scrambling of the summed Walsh encoded data symbols as well as isolation between groups of users and synchronization. PN encoding uses a long PN which is real followed by a short PN which is complex with real code components on the inphase and quadrature axes as shown in **4.** ~~Purpose of the separate PN encoding for the real and imaginary axes is to provide approximate orthogonality between the real and imaginary axes, since the same Walsh orthogonal codes are being used for these axes. Another PN encoding can be used as illustrated in these equations for the combined real and imaginary CDMA signals to provide scrambline and isolation between groups of users.~~

Receiver equations **(2)** describe a representative ~~real~~ Walsh CDMA decoding for the receiver in FIG. **3.** The receiver front end **5** provides estimates $\{\hat{Z}(n)=\hat{R}(n)+\mathrm{j}\,\hat{I}(n)\}$ of the transmitted real Walsh CDMA encoded chips $\{Z(n)=R(n)+jI(n)\}$ for the complex and real data symbols. Orthogonality property **6** is expressed as a matrix product of the ~~real~~ Walsh code chips or equivalently as a matrix ~~produce~~ product of the Walsh code chip numerical signs. ~~The 2-phase PN codes 7 have the useful decoding property that the square of each code chip is unity which is equivalent to observing that the square of each code chip numerical sign is unity.~~ Decoding algorithms **8** perform the inverse of the signal processing for the encoding in equations **(1)** to recover estimates $\{\hat{Z}(u)\}$ or $\{\hat{R}(u_R),\ \hat{I}(u_I)\}$ of the transmitter user symbols $\{Z(u)\}$ or $\{R(u_R),\ I(u_I)\}$ for the respective complex or real data symbols.

Current real Walsh CDMA decoding for receiver **(2)**

5 Receiver front end provides estimates $\{\hat{Z}(n) = \hat{R}(n) + j\hat{I}(n)\}$ of the encoded transmitter chip symbols $\{Z(n)=R(n)+jI(n)\}$ for the complex and real data symbols

6 Orthogonality property of ~~real~~ Walsh NxN matrix W

$$\sum_n W(\hat{u},n)W(n,u) = \sum_n \text{sign}\{W(\hat{u},n)\}\,\text{sign}\{W(n,u)\}$$

$$= \mathbf{N}\ \delta(\hat{u}, u)$$

where $\delta(\hat{u}, u)$ = Delta function of $\hat{u}$ and u

$= 1$ for $\hat{u} = u$

$= 0$ otherwise

7 PN decoding property:

$P_2(n)P_2(n) = \text{sgn}\{P_2(n)\ \text{sgn}\{P_2(n)\}$

$= 1$

Decoding algorithm:

Complex data symbols

$$\hat{Z}(u) =$$

$$2^{-1}\mathbf{N}^{-1}\sum_{\mathbf{n}} \hat{\mathbf{Z}}(\mathbf{n})[\text{sign}\{P_2(n)\}\ [\text{sign}\{P_R(n)\} - j\,\text{sgn}\{P_I(n)\}]\ \text{sgn}\{W(n,u)\}$$

= Receiver estimate of the transmitted complex data symbol Z(u)

Real data symbols

$$\hat{R}(u_R) =$$

$$\text{Real}[\ 2^{-1}\mathbf{N}^{-1}\sum_{\mathbf{n}}\hat{\mathbf{Z}}(\mathbf{n})[\text{sgn}\{P_R(n)\} - j\,\text{sgn}\{P_I(n)\}]\ \text{sgn}\{P_2(n)\}\text{sgn}\{W(n, u_R)\}$$

= Receiver estimate of the transmitted complex data symbol $R(u_R)$

$$\hat{I}(u_I) =$$

$$\mathrm{Imag}[\ 2^{-1}N^{-1}\sum_{n}\hat{Z}(\mathbf{n})[\mathrm{sgn}\{P_R(n)\}-j\,\mathrm{sgn}\{P_I(n)\}]\ \mathrm{sgn}\{P_2(n)\}\mathrm{sgn}\{W(n,u_I)\}$$

= Receiver estimate of the transmitted complex data symbol $I(u_I)$

FIG. **1** CDMA transmitter block diagram is representative of a current CDMA transmitter which includes an implementation of the current real Walsh CDMA channelization encoding in equations **(1)**. This block diagram becomes a representative implementation of the CDMA transmitter which implements the ~~new~~ complex Walsh CDMA encoding of this invention disclosure when the current real Walsh CDMA encoding **13** is replaced by the ~~new~~ complex Walsh CDMA encoding ~~of this invention~~. Signal processing starts with the stream of user input data words **9**. Frame processor **10** accepts these data words and performs the encoding and frame formatting, and passes the outputs to the symbol encoder **11** which encodes the frame symbols into amplitude and phase coded symbols **12** which could be complex $\{Z(u)\}$ or real $\{R(u_R), I(u_I)\}$ depending on the application. These symbols **12** are the inputs to the current real Walsh CDMA encoding in equations **(1)**. Inputs $\{Z(u)\}$, $\{R(u_R), I(u_I)\}$ **12** are ~~real~~ Walsh encoded, summed over the users, and scrambled by PN in the current ~~real~~ Walsh CDMA encoder **13** to generate the complex output chips $\{Z(n)\}$ **14**. This encoding **13** is a representative implementation of equations **(1)**. These output chips $Z(n)$ are waveform modulated **15** to generate the analog complex signal $z(t)$ which is single sideband upconverted, amplified, and transmitted (Tx) by the analog front end of the transmitter **15** as the real waveform $v(t)$ **16** at the carrier frequency f_0 whose amplitude is the real part of the complex envelope of the baseband waveform $z(t)$ multiplied by the carrier frequency and the phase angle ϕ accounts for the phase change from the baseband signal to the transmitted signal.

~~It should be obvious to anyone skilled in the communications art that this example implementation in FIG. 1 clearly defines the fundamental CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches.~~

FIG. **2** real Walsh CDMA encoding is a representative implementation of the ~~real~~ Walsh CDMA encoding **13** in FIG. **1** and in equations **(1)**. Inputs are the user data symbols which could be complex $\{Z(u)\}$ or real $\{R(u_R)$, ~~+j~~$I(u_I)\}$ **17**. For complex and real data symbols the encoding of each user by the corresponding Walsh code is described in **18** by the implementation of transferring the sign of each Walsh code chip to the user data symbol followed by a 1-to-N expander $1\uparrow N$ of each data symbol into an N chip sequence using the sign transfer of the Walsh chips.

For complex data symbols $\{Z(u)\}$ the sign-expander operation **18** generates the N-chip sequence $Z(u,n) = Z(u)\mathrm{sgn}\{W(u,n)\} = Z(u)W(u,n)$ for $n=0,1,\ldots,N-1$ for each user $u=0,1,\ldots,N-1$. This Walsh encoding serves to spread each user data symbol into an orthogonally encoded chip sequence which is spread over the CDMA communications frequency band. The Walsh encoded chip sequences for each of the user data symbols are summed over the users **19** ~~followed~~ and encoded ~~by PN encoding~~ with a long code $P_2(n)$ ~~with~~ followed by a short code ~~the scrambling sequences~~ $[P_R(n)+jP_I(n)]$ **21**. ~~PN encoding is implemented by transferring the sign of each PN chip to the summed chip of the Walsh encoded data symbols.~~ Output is the stream of complex CDMA encoded chips $\{Z(n)\}$ **22**. The switch **20** selects the appropriate signal processing path for the complex and real data symbols.

For real data symbols $\{R(u_R)$, ~~+j~~$I(u_I)\}$ the real and imaginary communications axis data symbols are separately Walsh

encoded **18**, summed **19**, and then PN encoded **19** with long codes $P_{R2}(n)$ for the real axis and $P_{I2}(n)$ for the imaginary axis to provide orthogonality between the channels along the real and imaginary communications axes. Output is complex combined **19** and PN encoded with the ~~scrambling~~ short PN sequence $[P_R(n)+jP_I(n)]$ **21**. Output is the stream of complex CDMA encoded chips $\{Z(n)\}$ **22**.

~~It should be obvious to anyone skilled in the communications art that this example implementation in FIG. 2 clearly defines the fundamental CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches.~~

FIG. **3** CDMA receiver block diagram is representative of a current CDMA receiver which includes an implementation of the current real Walsh CDMA decoding in equations **(2)**. This block diagram becomes a representative implementation of the CDMA receiver which implements the ~~new~~ complex Walsh CDMA decoding when the current real Walsh CDMA decoding **27** is replaced by the ~~new~~ complex Walsh CDMA decoding of this invention. FIG. **3** signal processing starts with the user transmitted wavefronts incident at the receiver antenna **23** for the n_u users $u = 1,\ldots, n_u \le N_c$. These wavefronts are combined by addition in the antenna to form the receive (Rx) signal $\hat{v}(t)$ at the antenna output **23** where $\hat{v}(t)$ is an estimate of the transmitted signal v(t) **16** in FIG. **1**, that is received with errors in time Δt, frequency Δf, phase $\Delta\theta$, and with an estimate $\hat{z}(t)$ of the transmitted complex baseband signal z(t) **16** in FIG. **1**. This received signal $\hat{v}(t)$ is amplified and downconverted by the analog front end **24** and then synchronized (synch.) and analog-to-digital (ADC) converted **25**. Outputs from the ADC are filtered and chip detected **26** by the

fullband chip detector, to recover estimates $\{\hat{Z}(n) = \hat{R}(n) + j\,\hat{I}(n)\}$ **28** of the transmitted signal which is the stream of complex CDMA encoded chips $\{Z(n)=R(n)+jI(n)\}$ **14** in FIG. **1** for both complex and real data symbols. The CDMA decoder **27** implements the algorithms in equations **(2)** by stripping off the PN code~~(s)~~s and decoding the received CDMA real Walsh orthogonally encoded chips to recover estimates $\{\hat{Z}(u) = \hat{R}(u_R)+j\hat{I}(u_I)\}$ **29** of the transmitted user data symbols $\{Z(u)= R(u_R) + jI(u_I)\}$ **12** in FIG. **1**. Notation introduced in FIG. **1** and **3** assumes that the user index $u=u_R=u_I$ for complex data symbols, and for real data symbols the user index u is used for counting the user pairs (u_R, u_I) of real and complex data symbols. These estimates are processed by the symbol decoder **30** and the frame processor **31** to recover estimates **32** of the transmitted user data words.

~~It should be obvious to anyone skilled in the communications art that this example implementation clearly defines the fundamental current CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches.~~

FIG. **4** real Walsh CDMA decoding is a representative implementation of the real Walsh CDMA decoding **27** in FIG. **3** and in equations **(2)**. Inputs are the received estimates of the complex CDMA encoded chips $\{\hat{Z}(n)\}$ **33**. The PN ~~scrambling~~ codes ~~is~~ are stripped off from these chips **34** by ~~changing the sign of each chip according to~~multiplying by the numerical sign of the real and imaginary components of the complex conjugate of the PN code as per the decoding algorithms ~~8~~ 7 in equations **(2)**.

For complex data symbols **35** the long PN code is stripped off and the real Walsh channelization coding is removed by a

pulse compression operation consisting of multiplying each received chip by the numerical sign of the corresponding Walsh chip for the user, scaling by 1/2N, and summing the products over the N Walsh chips **36** to recover estimates $\{\hat{Z}(u)\}$ of the user complex data symbols $\{Z(u)\}$. The switch **35** selects the appropriate signal processing path for the complex and real data symbols.

For real data symbols **35** the next signal processing operation is the removal of the remaining PN codes from the real and imaginary axes. This is followed by stripping off the ~~real~~ Walsh channelization coding by multiplying each received chip by the numerical sign of the corresponding Walsh chip for the user, scaling by 1/2N, and summing the products over the N Walsh chips **36** to recover estimates $\{\hat{R}(u_R), \hat{I}(u_I)\}$ of the user real data symbols $\{R(u_R), I(u_I)\}$.

It should be obvious to anyone skilled in the communications art that ~~this~~ these example implementations clearly define~~s~~ the fundamental current CDMA signal processing relevant to this invention disclosure and it is obvious that ~~this~~ these examples ~~is~~ are representative of the other possible signal processing approaches.

For cellular applications the transmitter description describes the transmission signal processing applicable ~~to~~ of this invention for both the hub and user terminals, and the receiver describes the corresponding receiving signal processing for the hub and user terminals for applicability ~~to~~ of this invention.

SUMMARY OF THE INVENTION

~~SUMMARY OF INVENTION~~

This invention is a new approach to the application of Walsh orthogonal codes for CDMA, which offers to replaces the current real Walsh codes with ~~the new~~ complex Walsh codes called hybrid Walsh codes and ~~the hybrid~~ generalized complex Walsh codes called generalized hybrid Walsh codes. ~~disclosed in this invention.~~ Real Walsh codes are used for current CDMA applications ~~and will be used for all of the future CDMA systems. This invention of~~ and complex Walsh codes will provide the choice of using ~~the new~~ complex Walsh codes or the real Walsh codes since the permutated real Walsh codes are the real components of the complex Walsh codes. This means an application capable of using the complex Walsh codes can simply turn-off the complex axis components of the complex Walsh codes for real Walsh CDMA coding and decoding.

The complex Walsh codes of this invention are proven to be the natural development for the Walsh codes and therefore are the correct complex Walsh codes to within arbitrary factors that include scale and rotation, which are not relevant to performance. This natural development of the complex Walsh codes in the N-dimensional complex code space C^N extended the correspondences between the real Walsh codes and the Fourier codes in the N-dimensional real code space R^N, to correspondences between the complex Walsh codes and the discrete Fourier transform (DFT) complex codes in C^N.

These ~~new~~ 4-phase complex Walsh orthogonal CDMA codes provide fundamental performance improvements compared to the 2-phase real Walsh codes which include an increase in the carrier-to-noise ratio (CNR) for data symbol recovery in the receiver, lower correlation side-lobes under timing offsets both with and

without PN spreading, lower levels of harmonic interference caused by non-linear amplification of multi-carrier CDMA signals, and reduced phase tracking jitter for code tracking to support both acquisition and synchronization. These potential performace improvements simply reflect the widely known principle that complex CDMA is better than real CDMA.

The ~~new hybrid~~ gemeralized complex Walsh orthogonal CDMA codes increase the choices for the code length by allowing the combined use of ~~complex~~ hybrid Walsh, Walsh, and discrete Fourier transform complex orthogonal codes using a Kronecker or tensor cconstruction, direct sum constructio~~i~~n, as well as the possibility for more general functional combining.

~~**BRIEF DESCRIPTION OF DRAWINGS AND PERFORMACE DATA**~~

BRIEF DESCRIPTION OF THE DRAWINGS AND THE PERFORMANCE DATA

The above-mentioned and other features, objects, design algorithms, implementations, and performance advantages of the present invention will become more apparent from the detailed description set forth below when taken in conjunction with the drawings and performance data wherein like reference characters and numerals denote like elements, and in which:

FIG. **1** is a representative CDMA transmitter signal processing implementation block diagram~~,~~ with emphasis on the current real Walsh CDMA encoding which contains the signal processing elements addressed by this invention disclosure.

FIG. **2** is a representative CDMA encoding signal processing implementation diagram with emphasis on the current real Walsh

CDMA encoding which contains the signal processing elements addressed by this invention disclosure.

FIG. **3** is a representative CDMA receiver signal processing implementation block diagram~~,~~ with emphasis on the current real Walsh CDMA decoding which contains the signal processing elements addressed by this invention disclosure.

FIG. **4** is a representative CDMA decoding signal processing implementation diagram~~,~~ with emphasis on the current real Walsh CDMA decoding which contains the signal processing elements addressed by this invention disclosure.

FIG. **5** is a representative correlation plot of the correlation between the complex discrete Fourier transform (DFT) cosine and sine code component vectors and the real Fourier transform cosine and sine code component vectors.

FIG. **6** is a representative CDMA encoding signal processing implementation diagram with emphasis on the ~~new complex~~ hybrid Walsh CDMA encoding which contains the signal processing elements addressed by this invention disclosure

FIG. **7** is a representative CDMA decoding signal processing implementation diagram with emphasis on the ~~new~~ hybrid Walsh CDMA decoding which contains the signal processing elements addressed by this invention disclosure.

~~DISCLOSURE OF INVENTION~~

DISCLOSURE OF THE INVENTION

Consider the ~~Real~~ real orthogonal CDMA code space R^N for Hadamard, Walsh, and Fourier codes~~:~~ . The new complex Walsh orthogonal CDMA codes are called hybrid Walsh codes and are derived from the current real Walsh codes by starting with the correspondence of the current real Walsh codes with the discrete real Fourier transform ~~(DFT)~~ basis vectors. ~~Consider the real orthogonal CDMA code space R^N consisting of N-orthogonal real code vectors.~~ Examples of code sets in R^N consisting of N-orthogonal real code vectors include the Hadamard, Walsh, and Fourier. The corresponding matrices of code vectors are designated as H, W, F respectively and as defined in equation~~s~~ **(~~1~~3)** ~~respectively~~ consist of N-rows of N-chip code vectors. Hadamard codes in their re-ordered form known as Walsh codes are used in the current CDMA, in the proposals for the next generation G3 CDMA, and in the proposals for all future CDMA. Walsh codes re-order the Hadamard codes according to increasing sequency. ~~These codes assumed +/-1 values.~~ Sequency is the average rate of change of the sign of the codes. ~~and the reordering places the Walsh codes in correspondence to the DFT wherein sequency is in correspondence with frequency in the DFT.~~

~~It is important to note that the correspondence "sequency-frequency" only applies to the complex DFT matrix E consisting of the N-row vectors {E(u)= [E(u,0),…,E(u,N-1)} wherein the elements of E are~~ $\cancel{E(u,n)=e^{j(2\pi\, un/N)},\ u,n = 0,1,\ldots,N-1\}}$~~. Historically it has not been applied to the Fourier basis F in~~ $\cancel{R^N}$~~.~~

Equation~~s~~ **(~~1~~3)** define the three sets H,W,F of real orthogonal codes in R^N with the understanding that the H and W are identical except for the ordering of the code vectors.

Hadamard **37** and Walsh **38** orthogonal functions are basis vectors in R^N and are used as code vectors for orthogonal CDMA channelization coding. Hadamard **37** and Walsh **38** equations of definition are widely known. ~~with examples given in Reference [6].~~ Likewise, the Fourier **39** equations of definition are widely known within the engineering and scientific communities, wherein

N-chip real orthogonal CDMA codes **(3)**

37 Hadamard codes

H = Hadamard NxN orthogonal code matrix consisting of N rows of N chip code vectors

= [H(u)] matrix of row vectors H(u)

= [H(u,n)] matrix of elements H(u,n)

H(u) = Hadamard code vector u

= [H(u,0), H(u,1), …, H(u,N-1)]

= 1xN row vector of chips H(u,0),….,H(u,N-1)

H(u,n) = Hadamard code u chip n

= +/+1 possible values

$$= (-1)\wedge\left[\sum_{i=0}^{i=M-1} u_i n_i\right]$$

where $u = \sum_{i=0}^{i=M-1} u_i 2^i$ binary representation of u

$n = \sum_{i=0}^{i=M-1} n_i 2^i$ binary representation of n

38 Walsh codes

W = Walsh NxN orthogonal code matrix consisting of N rows of N chip code vectors

= [W(u)] matrix of row vectors W(u)

= [W(u,n)] matrix of elements W(u,n)

W(u) =Walsh code vector u

= [W(u,0), W(u,1), …, W(u,N-1)]

W(u,n) = Walsh code u chip n

= +/+1 possible values

$$= (-1)\hat{}\ [u_{M-1}n_0 + \sum_{i=1}^{i=M-1}(u_{M-1-i} + u_{M-i})n_i\ ]$$

39 Fourier codes

F = Fourier NxN orthogonal code matrix consisting of N rows of N chip code vectors

= [F(u)] matrix of row vectors F(u)

$$= \begin{bmatrix} C \\ \hline S \end{bmatrix}$$

C = N/2+1 x N matrix of row vectors C(u)

C(u) = Even code vectors for u=0,1,…,N/2

$= [1,\ \cos(2\pi\ u1/N),\ldots,\cos(2\pi\ u\cancel{(N-1}/\cancel{N}\underline{2})]$

S = N/2-1 x N matrix of row vectors S(u)

S(Δu) = Odd code vectors for u=N/2+Δu, Δu=1,2,…,N/2-1

$= [\sin(2\pi\ \Delta u\ 1/N),\ldots,\sin(2\pi\ \Delta u(\cancel{N}\underline{1/2}-1\cancel{)}/N)\underline{)}]$

where F(u) = C(u) for u=0,1,…,N/2

= S(Δu) for Δu = u-N/2, u=N/2+1,…,N-1

the cosine C(u) and sine S(u) code vectors are the code vectors of the Fourier code matrix F.

~~Complex~~ Consider the complex orthogonal CDMA code space C^N for DFT codes~~:~~. The DFT orthogonal codes are a complex basis for the complex N-dimensional CDMA code space C^N and consist of the DFT harmonic code vectors arranged in increasing order of frequency. Equations **(4)** are the definition of the DFT code vectors. The DFT definition **40** is widely known within the

engineering and scientific communities. Even and odd components of the DFT code vectors **41** are the real cosine code vectors {C(u)} and the imaginary sine code vectors (S(u)} where even and odd are referenced to the midpoint of the code vectors. These cosine and sine code vectors are the extended set 2N of the N Fourier cosine and sine code vectors.

N-chip DFT complex orthogonal CDMA codes ________ **(4)**

40 DFT code vectors

E = DFT NxN orthogonal code matrix consisting of N rows of N chip code vectors

= [E(u)] matrix of row vectors E(u)

= [E(u,n)] matrix of elements E(u,n)

E(u) = DFT code vector u

= [E(u,0), E(u,1), …, E(u,N-1)]

= 1xN row vector of chips E(u,0),….,E(u,N-1)

E(u,n) = DFT code u chip n

= $e^{j2\pi\ un/N}$

= cos(2π un/N) + jsin(2π un/N)

= N possible values on the unit circle

41 Even and odd code vectors are the extended set of Fourier even and odd code vectors in **39** equations **(3)**

C(u) = Even code vectors for u=0,1,…,N-1

= [1, cos(2π u 1/N),…,cos(2π u (N-1)/N)]

S(u) = Odd code vectors for u=0,1,…,N-1

= [0, sin(2π u 1/N),…,sin(2π u (N-1)/N)]

E(u) = C(u) + j S(u) for u=0,1,…,N-1

Consider the ~~Complex~~ complex orthogonal CDMA code space C^N for ~~complex~~hybrid Walsh codes~~:~~. Step **1** in the derivation of

the ~~complex~~hybrid Walsh codes in this invention establishes the correspondence of the even and odd Walsh codes with the even and odd Fourier codes. Even and odd for these codes are with respect to the midpoint of the row vectors similar to the definition for the DFT vector codes **41** in equations **(4)**. Equations **(5)** identify the even and odd Walsh codes in the W basis in R^N. These even and odd Walsh codes can be placed in

Even and odd Walsh codes in R^N **(5)**

$$
\begin{aligned}
W_e(u) &= \text{Even Walsh code vector} \\
&= W(2u) \quad \text{for } u=0,1,\ldots,N/2-1 \\
W_o(u) &= \text{Odd Walsh code vectors} \\
&= W(2u-1) \quad \text{for } u=1,\ldots,N/2
\end{aligned}
$$

direct correspondence with the Fourier code vectors **39** in equations **(3)** using the DFT equations **(4)**. This correspondence is defined in equations **(6)** where the correspondence operator "~" represents the even and odd correspondence between the Walsh and Fourier codes, and additionally represents the sequency~frequency correspondence.

Correspondence between Walsh and Fourier codes **(6)**

$$
\begin{aligned}
W(0) &\sim C(0) \\
W_e(u) &\sim C(u) \quad \text{for } u=1,\ldots,N/2-1 \\
W_o(u) &\sim S(u) \quad \text{for } u=1,\ldots,N/2-1 \\
W(N-1) &\sim C(N/2)
\end{aligned}
$$

Step **2** derives the set of N complex DFT vector codes in C^N from the set of N real Fourier vector codes in R^N. This means that the set of 2N cosine and sine code vectors in **41** in equations **(4)** for the DFT codes in C^N will be derived from the

set of N cosine and sine code vectors in **39** in equations **(3)** for the Fourier codes in R^N. The first N/2+1 code vectors of the DFT basis can be written in terms of the Fourier code vectors in equations **(7)**.

DFT code vectors 0,1,…,N/2 derived from Fourier ______**(7)**

42 Fourier code vectors from **39** in equations **(3)** are

$C(u)$ = Even code vectors for $u=0,1,\ldots,N/2$

$= [1, \cos(2\pi\ u1/N),\ldots,\cos(2\pi\ u\cancel{(N-1/N}\underline{/2})]$

$S(u)$ = Odd code vectors for $u=1,2,\ldots,N/2-1$

$= [\sin(2\pi\ u1/N),\ldots,\sin(2\pi\ u(\cancel{N}\underline{1/2}-1\cancel{)}/N)\underline{)}]$

43 DFT code vectors in **41** of equations (4) are written as functions of the Fourier code vectors

$E(u)$ = DFT complex code vectors for $u=0,1,\ldots,N/2$

$= C(0)$

$= C(u)+jS(u)$ for $u=1,\ldots,N/2-1$

$= C(N/2)$ for $u=N/2$

The remaining set of N/2+1,…,N-1 DFT code vectors in C^N can be derived from the original set of Fourier code vectors by a correlation which establishes the mapping of the DFT codes onto the Fourier codes. We derive this mapping by correlating the real and imaginary components of the DFT code vectors with the corresponding even and odd components of the Fourier code vectors. The correlation operation is defined in equations **(8)**

Correlation of DFT and Fourier code vectors **(8)**

$\mathrm{Corr(even)} = C^*\mathrm{Real}\{E'\}$

= Correlation matrix

= Matrix product of C^* and the real part of E transpose

$$\text{Corr(odd)} = S^*\text{Imag}\{E'\}$$

= Correlation matrix

= Matrix product of S and the imaginary part of E transpose

wherein "*" is the matrix product, "'" is the connugate transpose operator, and the results of the correlation calculations are plotted in FIG. **5** for N=32 for the real cosine and the odd sine Fourier code vectors. Plotted are the correlation of the 2N DFT cosine and sine codes against the N Fourier cosine and sine codes which range from -15 to +16 where the negative indices of the codes represent a negative correlation value. The plotted curves are the correlation peaks. These correlation curves in FIG. **5** prove that the remaining N/2+1,…,N-1 code vectors of the DFT are derived from the Fourier code vectors by equations **(9)**. This

DFT code vectors N/2+1,…, N-1 derived from Fourier **(9)**

$$E(u) = C(N/2 - \Delta u) - jS(N/2 - \Delta u)$$

$$\text{for} \quad u = N/2 + \Delta u$$

$$\Delta u = 1, \ldots, N/2-1$$

~~This~~ construction of the remaining DFT basis in equations **(9)** is an application of the DFT spectral foldover property which observes the DFT harmonic vectors for frequencies $fNT=N/2+\Delta$~~i~~u above the Nyquist sampling rate $fNT=N/2$ simply foldover such that the DFT harmonic vector for $fNT = N/2+\Delta$~~i~~u is the DFT basis vector for $fNT = N/2-\Delta$~~i~~u to within a fixed sign and where "f" is frequency and "T" is sample interval.
~~fixed phase angle of rotation.~~

Step **3** derives the ~~complex~~ hybrid Walsh code vectors from the real Walsh code vectors by using the DFT derivation in equations **(7)** and **(9)**, by using the correspondences between the real Walsh and Fourier in equations **(6)**, and by using the fundamental correspondence between the ~~complex~~ hybrid Walsh and the complex DFT given in equation **(10)**.

Correspondence between ~~complex~~ hybrid Walsh and DFT **(10)**

$\tilde{W} \sim E$ = NxN complex DFT orthogonal code matrix

where

$\tilde{W}$ = NxN ~~complex~~ hybrid Walsh orthogonal code matrix

= N rows of N chip code vectors

= [$\tilde{W}(u)$] matrix of row vectors $\tilde{W}(u)$

= [$\tilde{W}(u,n)$] matrix of elements $\tilde{W}(u,n)$

$\tilde{W}(u)$ = ~~Complex~~ Hybrid Walsh code vector u

= [$\tilde{W}(u,0)$, $\tilde{W}(u,1)$, …, $\tilde{W}(u,N-1)$]

$\tilde{W}$ = +/-1 +/- j possible value

We start by constructing the complex Walsh dc code vector $\tilde{W}(0)$. We use equation $E(0)=C(0)$ in **43** in equations **(7)**,

the correspondence in equations **(6)**, and observe that the dc ~~complex~~ hybrid Walsh vector has both real and imaginary components in the $\widetilde{W}$ domain, to derive the dc ~~complex~~ hybrid Walsh code vector equation:

$$\widetilde{W}(0) = W(0) + jW(0) \qquad \text{for } u=0 \qquad \textbf{(11)}$$

For ~~complex~~ hybrid Walsh code vectors $\widetilde{W}(u)$, u=1,2,…,N/2-1, we start with the Walsh code properties in **(5)**,**(6)** and apply the correspondences in equations **(10)** between the ~~complex~~ hybrid Walsh and DFT bases, to the DFT equations **43** in equations **(7)** ~~:~~) to derive the equations:

$$\begin{aligned}\widetilde{W}(u) &= W_e(u) + jW_o(u) && \text{for } u=1,2,\ldots,N/2-1 \qquad \textbf{(12)}\\ &= W(2u) + jW(2u-1) && \text{for } u=1,2,\ldots,N/2-1\end{aligned}$$

For ~~complex~~ hybrid Walsh code vector $\widetilde{W}(N/2)$ we use the equation E(N/2)=C(N/2) **43** in equations **(7)** and the same rationale used to derive equation **(11)**, to ~~yield~~ derive the equation: ~~for .~~

$$\widetilde{W}(u) = W(N-1) + jW(N-1) \qquad \text{for } u=N/2 \qquad \textbf{(13)}$$

For ~~complex~~ hybrid Walsh code vectors $\widetilde{W}(N/2+\Delta u)$, Δu=1,2,…,N/2-1 we apply the correspondences between the ~~complex~~ hybrid Walsh and DFT bases to the spectral foldover equation E(N/2+Δu)=C(N/2-Δu)-jS(N/2-Δu) in equation~~s~~ **(9)** with the changes in indexing required to account for the W indexing in equations **(5)**, **(6)** ~~. The~~ to derive the equations: ~~are~~

$$\tilde{W}(N/2+\Delta u) = W(N-1-\Delta eu)+W(N-1-\Delta ou) \qquad \mathbf{(14)}$$

$$\text{for } u=N/2+1,\ldots,N-1 \qquad \cancel{\mathbf{(14)}}$$

$$= W(N-1-2\Delta u) + jW(N-2\Delta u)$$

$$\text{for } u=N/2+1,\ldots,N-1$$

using the notation Δ~~ei~~eu=2Δ~~i~~u, Δ~~oi~~ou=2Δ~~i~~u-1. These ~~complex~~ hyubrid Walsh code vectors in equations **(11)**, **(12)**, **(13)**, **(14)** are the equations of definition for the ~~complex~~ hybrid Walsh code vectors.

An equivalent way to derive the ~~complex~~ hybrid Walsh code vectors in C^N from the real Walsh basis in R^{2N} is to use a sampling technique which is a known method for deriving a complex basis in C^N from a real basis in ~~R^N~~ R^{2N}.

Transmitter equations **(15)** describe a representative ~~complex~~ hybrid Walsh CDMA encoding for the transmitter in FIG. **1** It is assumed that there are N ~~complex~~ hybrid Walsh code vectors $\tilde{W}(u)$ **44** which are the ~~each of length N chips similar to the definitions for the real Walsh code vectors **1** in equations **(1)**. The code vector is presented by a~~ 1xN ~~N-chip row vector~~ ~~$\tilde{W}(u) = [\tilde{W}(u,0),\ldots,\tilde{W}(u,N-1)]$ where $\tilde{W}(u,n)$ is chip n of code u. The code vectors are the~~ row vectors of the ~~complex~~ hybrid Walsh matrix $\tilde{W}$. ~~Walsh code chip n of code vector u has the possible values $\tilde{W}(u,n)= +/-1 +/-j$.~~ Each user is assigned a unique Walsh code which allows the code vectors to be designated by the user symbols u=0,1,…,N-1 for N ~~complex~~ hybrid Walsh codes. The ~~complex~~ hybrid Walsh code vectors $\tilde{W}(u)$ derived in equations **(11)**,**(12)**,**(13)**,**(14)** are summarized **44** in terms of their real and imaginary component code vectors $\tilde{W}(u)=W_R(u)+jW_I(u)$ where $W_R(u)$ and $W_I(u)$ are respectively the real and imaginary component code vectors. As per the derivation of $\tilde{W}(u)$ the sets

of real axis code vectors $\{W_R(u)\}$ and the imaginary axis code vectors $\{W_I(u)\}$ both consist of the ~~real~~ Walsh code vectors in R^N with the ordering modified to ensure that the definition of the ~~complex~~ hybrid Walsh vectors satisfies equations **(11)**, **(12)**, **(13)**, **(14)**.

~~Complex~~ Hybrid Walsh CDMA encoding for transmitter **(15)**

44 ~~Complex~~ Hybrid Walsh codes ~~use the definitions for the real Walsh codes in 1 equations (1) and the definitions of the complex Walsh codes in equations~~ defined in **(11)**, **(12)**, **(13)**, **(14)** ~~We find~~

~~$\tilde{W}$ = complex Walsh NxN orthogonal code matrix consisting of N rows of N chip code vectors~~

~~= [$\tilde{W}$(u)] matrix of row vectors $\tilde{W}$(u)~~

~~= [$\tilde{W}$(u,n)] matrix of elements $\tilde{W}$(u,n)~~

$\tilde{W}(u)$ = ~~complex~~ Hybrid Walsh code vector u

$= W_R(u) + jW_I(u)$ for $u=0,1,\ldots,N-1$

where

$$
\begin{aligned}
W_R(u) &= \mathrm{Real}\{\ \tilde{W}(u)\ \} \\
&= W(0) && \text{for } u=0 \\
&= W(2u) && \text{for } u=1,2,\ldots,N/2-1 \\
&= W(N-1) && \text{for } u=N/2 \\
&= W(2N-2u-1) && \text{for } u=N/2+1,\ldots,N-1
\end{aligned}
$$

$$
\begin{aligned}
W_I(u) &= \mathrm{Imag}\{\ \tilde{W}(u)\ \} \\
&= W(0) && \text{for } u=0 \\
&= W(2u-1) && \text{for } u=1,2,\ldots,N/2-1 \\
&= W(N-1) && \text{for } u=N/2 \\
&= W(2N-2u) && \text{for } u=N/2+1,\ldots,N-1
\end{aligned}
$$

$\tilde{W}(u,n)$ = ~~complex~~ Hybrid Walsh code u chip n

= +/-1 +/-j possible values

45 Data symbols

$Z(u)$ = Complex data symbol for user u

$= R(u) + jI(u)$

46 ~~Complex~~ Hybrid Walsh encoded data

$$Z(u,n) = Z(u)\ \widetilde{W}(u,n)$$
$$= Z(u)[\mathrm{sgn}\{W_R(u,n)\} + j\mathrm{sgn}\{W_I(u,n)\}]$$
$$= [R(u)\mathrm{sgn}\{W_R(u,n)\} - I(u)\mathrm{sgn}\{W_I(u,n)\}]$$
$$+ j[R(u)\mathrm{sgn}\{W_I(u,n) + I(u,n)\mathrm{sgn}\{W_R(u,n)\}]$$

47 PN scrambling

~~$P_R(n)$ = Chip n of the PN code for the real axis~~

~~$P_I(n)$ = Chip n of the PN code for the imaginary axis~~

$Z(n)$ = PN scrambled complex Walsh encoded data chips after summing over the users

$$= \sum_u Z(u,n)\, P_2(n)\ [P_R(n) + j\ P_I(n)]$$

$$= \sum_u Z(u,n)\, \mathrm{sgn}\{P_2(n)\}\ [\mathrm{sgn}\{P_R(n)\} + j\ \mathrm{sgn}\{P_I(n)\}]$$

= ~~Complex~~ Hybrid Walsh CDMA encoded chips

User data symbols **45** are the set of complex symbols {Z(u), u=0,1…,N-1}. These data symbols are encoded by the hybrid Walsh CDMA codes **46**~~.~~ , ~~Each of the user symbols Z(u) is assigned a unique complex Walsh code $\widetilde{W}(u) = W_R(u) + jW_I(u)$. Complex Walsh encoding of each user data symbol generates an N-chip sequence with each chip in the sequence consisting of the user data symbol with the complex sign of the corresponding complex Walsh code chip, which means each encoded chip = [Data symbol Z(u)] x [Sign of $W_R(u)$ + j sign of $W_I(u)$].~~ ~~The complex Walsh encoded data symbols are summed and~~ encoded with PN scrambling codes comprising both long and short codes **47**~~.~~ , and summed over the users to yield the. complex hybrid Walsh CDMA encoded chips Z(n). ~~These PN codes are~~

~~defined 4 in equations **(1)** as a complex PN for each chip n, equal to [$P_R(u)$ + j $P_I(u)$] where $P_R(u)$ and $P_I(u)$ are the respective PN scrambling codes for the real and imaginary axes. Encoding with the complex PN is the same as given 4 in equations **(1)** for complex data symbols. Each complex Walsh encoded data chip Z(u,n) **46** is summed over the set of users u=0,1,…,N-1 and complex PN encoded to yield the complex Walsh CDMA chips Z(n) =~~ ~~$\sum_u Z(u,n)\, P_2(n)\, [P_R(n) + j\, P_I(n)]$ **47**.~~ Combinations of both real and complex data symbols can be used similar to the approach for the real Walsh in equations **(1)**.

~~Although not considered in this example, it is possible to use combinations of both complex and real data symbols similar to the approach for real Walsh CDMA encoding in equations **(1)** since the complex Walsh code vectors are the real Walsh code vectors along the real axis and a reordering of the real Walsh code vectors along theimaginary axis.~~

Receiver equations **(16)** describe a representative ~~complex~~ hybrid Walsh CDMA decoding for the receiver in FIG. **3**. The receiver front end **48** provides estimates $\{\hat{Z}(n)\}$ of the transmitted ~~complex~~ Walsh CDMA encoded chips {Z(n)} for the complex data symbols {Z(u)}. Orthogonality property **49** is expressed as a matrix product of the ~~complex~~ hybrid Walsh code chips or equivalently as a matrix ~~produce~~ of the ~~complex~~ hybrid Walsh code chip numerical signs of the real and imaginary components. ~~The 2-phase PN codes **50** have the useful decoding property that the square of each code chip is unity which is equivalent to observing that the square of each code chip numerical sign is unity.~~ Decoding algorithms **51** perform the inverse of the signal processing for the encoding in equations

(15) to recover estimates $\{\hat{Z}(u)\}$ of the transmitter user symbols {Z(n)} for the complex data symbols {Z(u)}. Combinations of both real and complex data symbols can be used similar to the approach for the real Walsh in equations **(2)**.

~~Complex~~ Hybrid Walsh CDMA decoding for receiver **(16)**

48 Receiver front end in FIG. **3** provides estimates $\{\hat{Z}(n)\}$ **28** of the encoded transmitter chip symbols {Z(n)} **47** in equations **(15)**.

49 Orthogoality property of ~~complex~~ Walsh NxN matrix $\widetilde{W}$

$$\sum_{\mathbf{n}} \widetilde{\mathbf{W}}(\hat{\mathbf{u}},\mathbf{n})\, \widetilde{\mathbf{W}}'(\mathbf{n},\mathbf{u}) =$$

$$\sum_n [\operatorname{sgn}\{W_R(\hat{u},n)\} + j\operatorname{sgn}\{W_I(\hat{u},n)\}][\operatorname{sgn}\{W_R(n,u) - j\operatorname{sgn}\{W_I(n,u)\}]$$

$$= 2N\ \delta(\hat{u}, u)$$

where $\delta(\hat{u}, u)$ = Delta function of $\hat{u}$ and u

= 1 for $\hat{u}$ = u

= 0 otherwise

~~**50** PN decoding property~~

~~P(n)P(n) = sgn{P(n)} sgn{P(n)}~~

~~= 1~~

51 Decoding algorithm

$$\hat{Z}(u) = 4^{-1}N^{-1} \sum_n \hat{Z}(n)\, sign\{P_2(n)\}\, [sign\{P_R(n)\} - j\ sign\{P_I(n)\}]^* [sign\{W_R(n,u)\} - j\ sign\{W_I(n,u)\}]$$

= Receiver estimate of the transmitted ~~data~~ data symbol Z(u) **45** in equations **(15)**

~~Although not considered in this example, it is possible to use combinations of both complex and real data symbols similar to the approach for real Walsh CDMA decoding in FIG. 4 since the complex Walsh code vectors are the real Walsh code vectors along the real axis and a reordering of the real Walsh code vectors along the imaginary axis~~

FIG. **6** ~~complex~~ hybrid Walsh CDMA encoding is a representative implementation of the ~~complex~~ hybrid Walsh CDMA encoding which will replace the current real Walsh encoding **13** in FIG. **1** and is defined in equations **(15)**. Inputs are the user data symbols $\{Z(u)\}$ **52.** Encoding of each user by the corresponding ~~complex~~ hybrid Walsh code implements the hybrid Walsh encoding in 46 in equations **(15)**. ~~is described in **53** by the implementation of transferring the sign +/-1+/-j of each complex Walsh code chip to the user data symbol followed by a 1-to-N expander 1↑N of each data symbol into an N chip sequence using the sign transfer of the complex Walsh chips. The sign-expander operation **53** generates the N-chip sequence~~ ~~$Z(u,n)=Z(u)[sgn\{W_R(u,n)\}+jsgn\{W_I(u,n)\}]= Z(u)[W_R(u,n)+jW_I(u,n)]$ for $n=0,1,\ldots,N-1$ for each user $u=0,1,\ldots,N-1$.~~

~~This complex Walsh encoding serves to spread each user data symbol into an orthogonally encoded chip sequence which is spread over the CDMA communications frequency band. The complex Walsh encoded~~ Encoded chip sequences for each of the user data symbols are summed over the users **54** followed by PN encoding with long and short codes ~~the scrambling sequence $[P_R(n)-jP_I(n)]$ **55**. PN encoding is implemented by transferring the sign of each PN chip to the summed chip of the Walsh encoded data symbols.~~ Output is the stream of complex CDMA encoded chips $\{Z(n)\}$ **56.** Combinations of both real and complex data symbols can be used similar to the approach for the real Walsh in FIG. **2.**

~~Although not considered in this example, it is possible to use combinations of both complex and real data symbols similar to the approach for real Walsh CDMA encoding in FIG. 2 since the complex Walsh code vectors are the real Walsh code vectors along the real axis and a reordering of the real Walsh code vectors along the imaginary axis.~~

~~It should be obvious to anyone skilled in the communications art that this example implementation in FIG. 6 clearly defines the fundamental CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches.~~

FIG. **7** ~~complex~~ hybrid Walsh CDMA decoding is a representative implementation of ~~complex~~ hybrid Walsh CDMA decoding which will replace the current real Walsh decoding **27** in FIG. **3**, and is defined in equations **(15)**. Inputs are the received estimates of the complex CDMA encoded chips $\{\hat{Z}(n)\}$ **57**. The PN scrambling code is stripped off from these chips **58** ~~by changing the sign of each chip according to the numerical sign of the real and imaginary components of the complex conjugate of the PN code as per~~ using the decoding algorithm~~s~~ ~~**50**~~ in equations **(16)**.

The ~~complex~~ hybrid Walsh channelization coding is removed ~~by a pulse compression operation consisting of multiplying each received chip by the numerical sign of the corresponding complex Walsh chip for the user~~**59** and the output scaled by 1/4N, and summing the products over the N Walsh chips **59** to recover estimates $\{\hat{Z}(u)\}$ of the user complex data symbols {Z(u)}. Combinations of both real and complex data symbols can be used similar to the approach for the real Walsh in FIG. **4**.

~~Although not considered in this example, it is possible to use combinations of both complex and real data symbols similar to the approach for real Walsh CDMA decoding in FIG. 4 since the complex Walsh code vectors are the real Walsh code vectors along the real axis and a reordering of the real Walsh code vectors along the imaginary axis.~~

It should be obvious to anyone skilled in the communications art that this example implementations in FIG. 6, 7 clearly defines~~s~~ the fundamental CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches.

For cellular applications the transmitter description describes the transmission signal processing applicable to this invention for both the hub and user terminals, and the receiver describes the corresponding receiving signal processing for the hub and user terminals for applicability to this invention.

Consider ~~Complex~~ complex orthogonal CDMA code space C^N for generalized hybrid ~~complex~~ Walsh codes~~: The power of 2 code lengths N=2^M where M is an integer, for complex Walsh can be modified to~~ which allow the code length N to be a product of powers of primes **60** in equations **(17)** or a sum of powers of primes **61** in equations **(17)**, at the implementation cost of introducing multiply operations into the CDMA encoding and decoding. In the previous disclosure of this invention ~~we used~~the N was assumed to be equal to a power of 2 which means N=2^m corresponding to prime p_0=2 and integer M=m_0. This restriction was made for convenience in explaining the construction of the ~~complex~~ hybrid Walsh and is not required since it is well known that Hadamard matrices exist for non-

integer powers of 2 and, therefore, ~~complex~~ hybrid Walsh matrices exist for non-integer powers of 2.

Length N of generalized hybrid Walsh ~~hybrid complex~~ codes **(17)**

60 Kronecker or tensor product code construction

$$N = \prod_{k} p_k \hat{} m_k$$

$$= \prod_{k} N_k$$

where

$\cancel{p_k}$ p_k = prime number indexed by k starting with k=0

m_k = order of the prime number p_k

N_k = Length of code for the prime p_k

$= \cancel{p_k^{m_k}}$ $p_k \hat{} m_k$

61 Direct sum code construction

$$N = \sum_{k} p_k \hat{} m_k$$

$$= \sum_{k} N_k$$

Add-only arithemetic operations are required for encoding and decoding both real Walsh and ~~complex~~ hybrid Walsh CDMA codes since the real Walsh values are +/-1 and the ~~complex~~ hybrid Walsh values are {+/-1 +/-j} or equivalently are {1,j,-1,-j} under a -90 degree rotation and normalization which means the only operations are sign transfers and adds plus subtracts ~~or~~ add-only algebraic operations. Multiply operations are more complex to implement than add operations. However, the advantages of having greater flexibility in choosing the orthogonal CDMA code lengths N using equations **(17)** can offset the expense of multiply operations for particular applications.

Accordingly, this invention includes the concept of generalized hybrid ~~hybrid complex~~ Walsh orthogonal CDMA codes with the flexibility to meet these needs. This extended class of ~~complex~~ hybrid Walsh codes ~~are hybrid in that their construction~~ supplements the ~~complex~~ hybrid Walsh codes ~~with the use of~~by combining with Hadamard, ~~(or~~ real Walsh~~)~~, DFT, and other orthogonal codes as well as with quasi-orthogonal PN by relaxing the orthogonality property to quasi-orthogonality.

Generalized ~~H~~hybrid ~~complex~~ Walsh orthogonal CDMA codes can be constructed as demonstrated in **64** and **65** in equations **(18)** for the Kronecker or tensor product, and in **66** ~~in equations (18)~~ for the direct sum. ~~Code~~ The example code matrices considered for orthogonal CDMA codes in **62** ~~in equations (18)~~ for the construction of the generalized hybrid ~~complex~~ Walsh are the DFT E and Hadamard H or equivalently Walsh W, in addition to the ~~complex~~ hybrid Walsh-$\widetilde{W}$. The algorithms and examples for the construction start with the definitions **63** of the NxN orthogonal code matrices $\widetilde{W}=\widetilde{W}_N$, , $E=E_N$, $H=H_N$ ~~for $\widetilde{W}$, E, H respectively, examples~~ for ~~low orders~~ N=2,4~~,~~ . and the equivalence of E_4 and $\widetilde{W}_4$ after the $\widetilde{W}_4$ is rotated through the angle ~~-90~~ 45 degrees and rescaled. The CDMA current and developing standards use the prime 2 which generates a code length $N=2^M$ where M=integer. For applications requiring greater flexibility in code length N, additional primes can be used using the ~~Kronecker~~ tensor construction. ~~We~~ This flexibility is illustrated ~~this~~ in **65** with the addition of prime=3. The use of prime=3 in addition to the prime=2 in the range of N=8 to N=64 is observed to increase the number of N choices ~~from 4 to 9~~ at a modest cost penal~~i~~ty of using multiples of the angle increment 30 degrees for prime=3 in addition to the angle increment 90 degrees for prime=2. As noted in **65** there are several choices in the ordering of the ~~Kronecker~~ tensor product construction and 2 of

these choices are used in the construction~~.~~ and these choices yield different sets of orthogonal codes.

Direct sum construction provides greater flexibility in the choice of N without necessarily introducing a multiply penality. However, the addition of the zero matrix in the construction is generally not desirable for CDMA communications.

Construction of generalized hybrid ~~hybrid complex~~ Walsh orthogonal codes **(18)**

62 Code matrices

$\widetilde{W}_N$ = NxN ~~complex~~ hybrid Walsh ~~orthogonal~~ code matrix

E_N = NxN DFT ~~orthogonal~~ code matrix

H_N = NxN Hadamard ~~orthogonal~~ code matrix

63 Low-order code definitions and equivalences

$$2\text{x}2 \quad H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= E_2$$

$$= (e^{-j\pi/4}/\sqrt{2}) * \widetilde{W}_2$$

$$3\text{x}3 \quad E_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{j2\pi/3} & e^{j2\pi 2/3} \\ 1 & e^{j2\pi 2/3} & e^{j2\pi/3} \end{bmatrix}$$

$$4x4 \quad H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$\widetilde{W}_4 = \begin{bmatrix} 1+j & 1+j & 1+j & 1+j \\ 1+j & -1+j & -1-j & 1-j \\ 1+j & -1-j & 1+j & -1-j \\ 1+j & 1-j & -1-j & -1+j \end{bmatrix}$$

$$E_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$= (e^{-j\pi/4}/\sqrt{2})\ \widetilde{W}_4$$

64 Kronecker or tensor product construction for $N=\prod_k N_k$

Code matrix C_N = NxN ~~hybrid orthogonal~~generalized hybrid Walsh CDMA code matrix

Kronecker or tensor product construction of C_N

$$\underline{C_N} = C_0 \prod_{k>0} \otimes C_{N_k}$$

Kronecker or tensor product definition

A = $N_a x N_a$ orthogonal code matrix [a_{ik}]

B = $N_b x N_b$ orthogonal code matrix

A⊗B = Kronecker or tensor product of matrix A

and matrix B

= N_aN_b x N_aN_b orthogonal code matrix consisting of the elements $[a_{ik}]$ of matrix A multiplied by the matrix B

= $[\ a_{ik}\ B\ ]$

65 Kronecker <u>or tensor </u>product construction examples for primes p=2,3 and the range of sizes $8 \le N \le 64$

8x8	C_8	$= \tilde{\mathbf{W}}_8$
12x12	C_{12}	$= \tilde{W}_4 \otimes E_3$
	C_{12}	$= E_3 \otimes \tilde{W}_4$
16x16	C_{16}	$= \tilde{W}_{16}$
18x18	C_{18}	$= \tilde{W}_2 \otimes E_3 \otimes E_3$
	C_{18}	$= E_3 \otimes E_3 \otimes \tilde{W}_2$
24x24	C_{24}	$= \tilde{W}_8 \otimes E_3$
	C_{24}	$= E_3 \otimes \tilde{W}_8$
32x32	C_{32}	$= \tilde{W}_{32}$
36x36	C_{36}	$= \tilde{W}_4 \otimes \tilde{W}_3 \otimes \tilde{W}_3$
	C_{36}	$= \tilde{W}_3 \otimes \tilde{W}_3 \otimes \tilde{W}_4$
48x48	C_{48}	$= \tilde{W}_{16} \otimes \tilde{W}_3$
	C_{48}	$= \tilde{W}_3 \otimes \tilde{W}_{16}$
64x64	C_{64}	$= \tilde{W}_{64}$

66 Direct sum construction for $N=\sum_k N_k$

Code matrix C_N = NxN hybrid orthogonal CDMA code matrix

Direct sum construction of C_N

$$C_N = C_0 \prod_{k>0} \oplus C_{N_k}$$

Direct sum definition

$A = N_a x N_a$ orthogonal code matrix

$B = N_b x N_b$ orthogonal code matrix

$A \oplus B$ = Direct sum of matrix A and matrix B

$= N_a+N_b \text{ x } N_a+N_b$ orthogonal code matrix

$$= \left[\begin{array}{c:c} A & O_{N_a x N_b} \\ \hdashline O_{N_b x N_a} & B \end{array}\right]$$

where $O_{N_1 x N_2} = N_1 x N_2$ zero matrix

It should be obvious to anyone skilled in the communications art that ~~this~~ these example implementations of the generalized hybrid ~~hybrid complex~~ Walsh in equations **(18)** clearly define~~s~~ the fundamental CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches. For example, the Kronecker or tensor product matrices E_N and H_N can be replaced by functionals.

For cellular applications the transmitter description which includes equations **(18)** describes the transmission signal processing applicable to this invention for both the hub and user terminals, and the receiver corresponding to the decoding of equations **(18)** describes the corresponding receiving signal processing for the hub and user terminals for applicability to this invention.

Consider ~~Computationally~~ computationally efficient encoding and decoding of complex Walsh CDMA codes and hybrid complex Walsh CDMA codes~~:~~. It is well known that fast and efficient encoding and decoding algorithms exist for the real Walsh CDMA codes. ~~These are documented in reference [6].~~ It is obvious that with suitable modifications these algorithms can be used to develop fast and efficient encoding and decoding algorithms for the ~~complex~~ hybrid Walsh CDMA codes since these complex codes have real and imaginary code vectors which are from the same set of real Walsh CDMA codes.

It is well known that the Kronecker or tensor product construction involving DFT, H and real Walsh orthogonal code vectors have efficient encoding and decoding algorithms. It is obvious that with suitable modifications these algorithms can be used to develop fast and efficient encoding and decoding algorithms for the Kronecker or tensor products of DFT, H and ~~complex~~ hybrid Walsh CDMA codes since these ~~complex~~ hybrid Walsh codes have real and imaginary code vectors which are from the same set of real Walsh CDMA codes. It is obvious that fast and efficient encoding and decoding algorithms exist for direct sum construction and functional combining.

Preferred embodiments in the previous description is provided to enable any person skilled in the art to make or use the present invention. The various modifications to these embodiments will be readily apparent to those skilled in the art, and the generic principles defined herein may be applied to other embodiments without the use of the inventive faculty. Thus, the present invention is not intended to be limited to the embodiments shown herein but is ~~not~~ to be accorded the wider scope consistent with the principles and novel features disclosed herein.

~~**REFERENCES:**~~

~~[1] IEEE Communications magazine July 2000 Vol. 38 No. 7, Multiple Access for Broadband Networks"~~

~~[2] IEEE Personal Communications April 1998 Vol. 5 No. 2, "Third Generation Mobile Systems in Europe"~~

~~[3] TIA/EIA interim standard, TIA/EIA/IS-95-A, May 1995~~

~~[4] United States Patent 5,715,236 Feb. 3 1998, "System and method for generating signal waveforms in a CDMA cellular telephone system"~~

~~[5] United Statees Patent 5,943,361 Aug 24 1999, "System and method for generating signal waveforms in a CDMA cellular telephone system"~~

~~[6] K.G. Beauchamp's book "Walsh functions and their Applications", Academic Press 1975~~

DRAWINGS AND PERFORMANCE DATA

FIG. 1 CDMA Transmitter Block Diagram

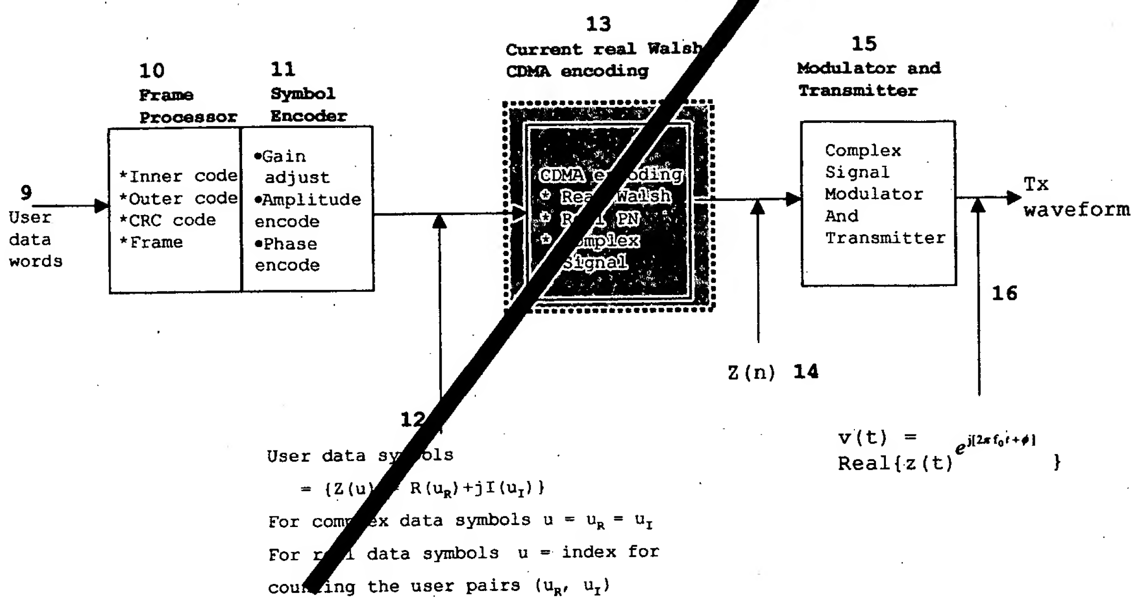

FIG. 2 Real Walsh CDMA Encoding

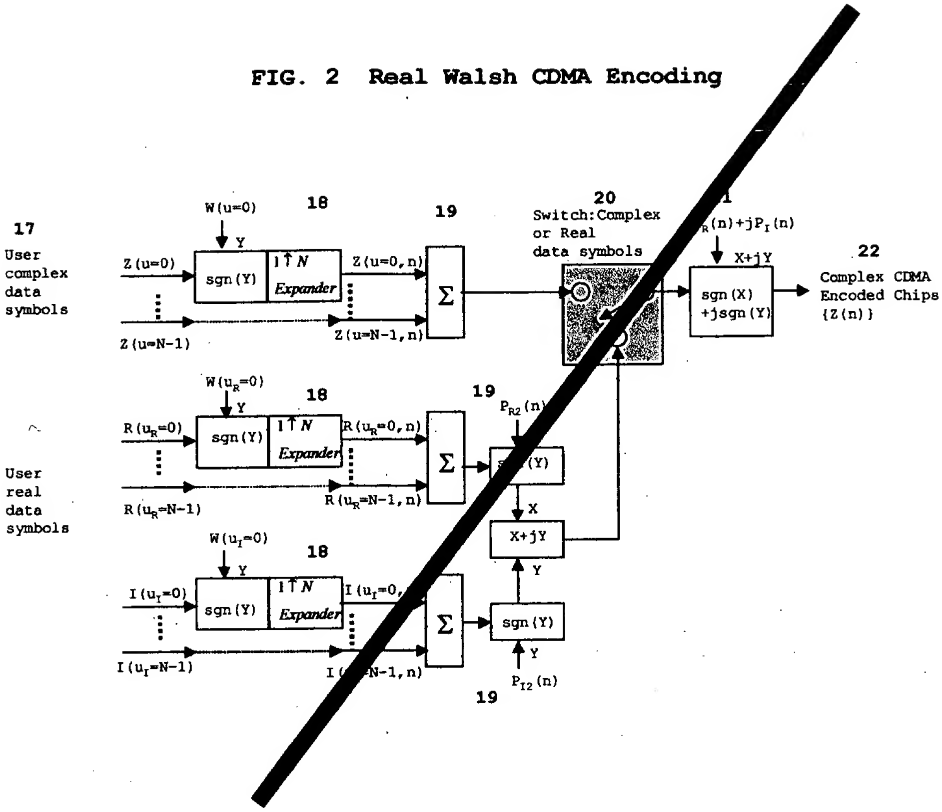

FIG. 3 CDMA Receiver Block Diagram

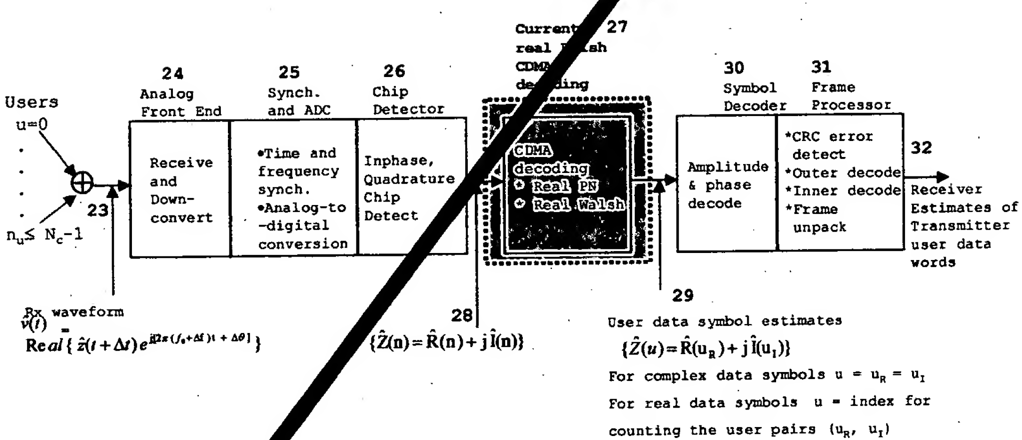

FIG. 4 Real Walsh CDMA Decoding

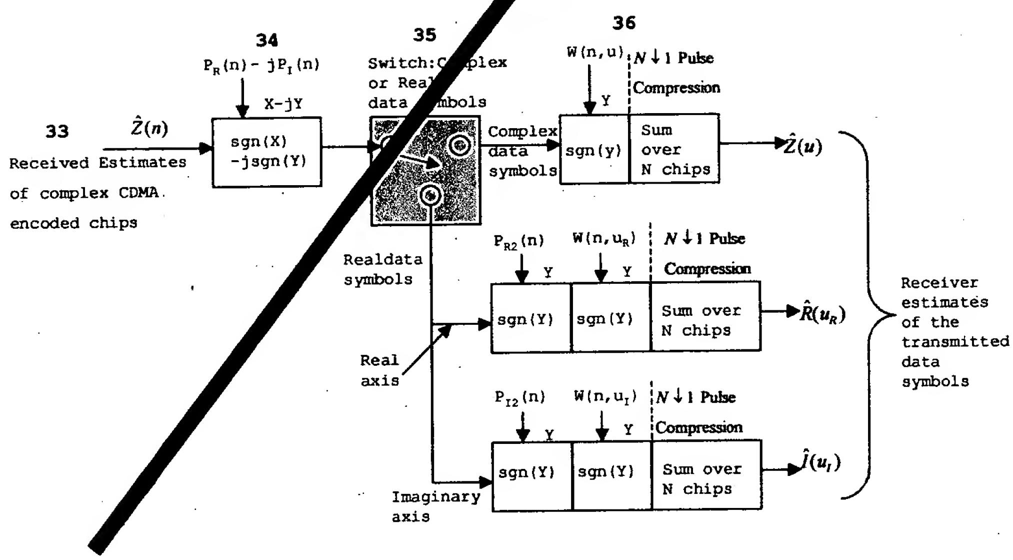

FIG. 5 Correlation of Fourier Codes with DFT Codes for N=32

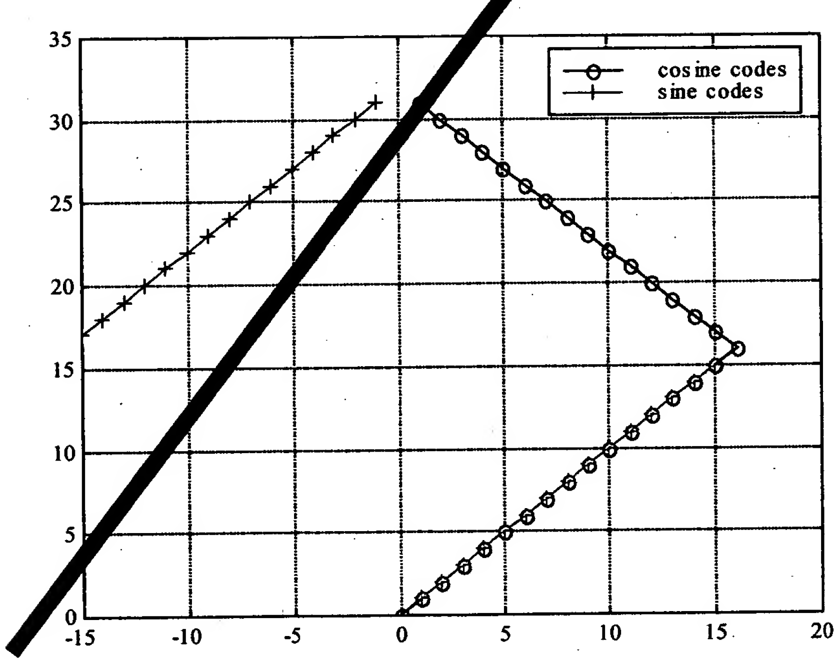

FIG. 6 Complex Walsh CDMA Encoding

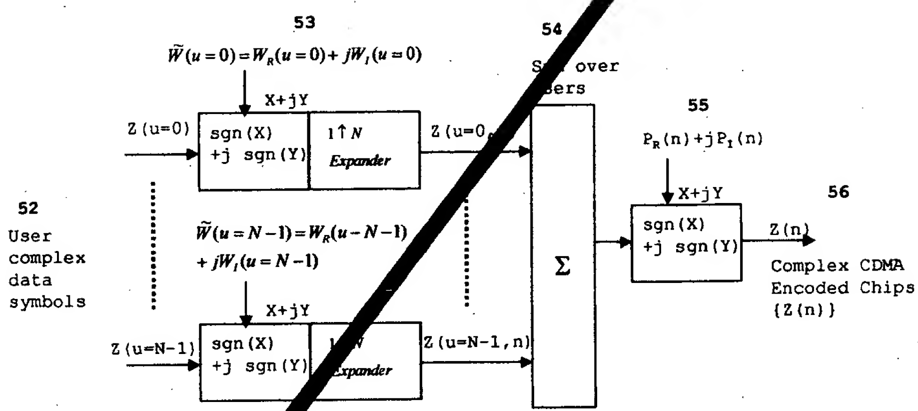

FIG. 7 Complex Walsh CDMA Decoding